Supersymmetric asymptotic safety is not guaranteed

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Abstract

It was recently shown that certain perturbatively accessible, non-supersymmetric gauge-Yukawa theories have UV asymptotic safety, without asymptotic freedom: the UV theory is an interacting RG fixed point, and the IR theory is free. We here investigate the possibility of asymptotic safety in supersymmetric theories, and use unitarity bounds, and the a-theorem, to rule it out in broad classes of theories. The arguments apply without assuming perturbation theory. Therefore, the UV completion of a non-asymptotically free susy theory must have additional, non-obvious degrees of freedom, such as those of an asymptotically free (perhaps magnetic dual) extension.

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Contents

I. INTRODUCTION

The discovery of the Higgs crowns the Standard Model as one of the most successful theories of Nature. Researchers have been desperately seeking hints of possible BSM extensions of the Higgs sector. Gauge-Yukawa theories are backbone of the Higgs sector, and it is therefore crucial to thoroughly investigate all possibilities for their dynamics.

Recently, a surprise was found. One can consider a gauge theory with too many matter fields to be asymptotically free in the UV, so it is instead infrared free. Perturbation theory suggests that the theory is UV unsafe (the Landau pole), requiring a UV cutoff or completion. A Yukawa interaction for the matter of this theory, on it's own, would also be IR-free and UV-unsafe. Taken together, however, at least for a range of colors and flavors, the individually unsafe gauge and Yukawa interactions can cure each other and combine together to lead to a perturbatively accessible, fully interacting, non-supersymmetric, RG fixed point in the ultraviolet [1]. See [1, 2] for the possibility that IR-free gauge and matter theory could have a UV fixed point even without the Yukawa interactions, albeit beyond perturbation theory.

The phenomenon of UV asymptotic safety opens the door to new model building possibilities [3, 4], and provide a novel playground to explore, e.g. the vacuum structure [5] and the thermodynamics of a new kind of matter [6]. The theories are different from the time-honoured case of *complete asymptotic freedom* [7–11], where the interactions instead shut off in the UV; see [12, 13] for recent studies.

Supersymmetry provides additional tools to explore the phases of gauge theories, beyond perturbation theory. For example, in $\mathcal{N} = 1$ duals, two UV asymptotically free theories can RG flow to the same IR SCFT, or an IR-free theory can be UV completed to an asymptotically free UV dual, with different gauge group and matter content [14] (see e.g. [15] for a review). In this latter case, the IR free theory avoids the Landau pole by completing to a different free theory in the UV; this is not the same as interacting asymptotic safety.

It is interesting to investigate if asymptotic safety can also occur in supersymmetric theories – both for model building and for better understanding the phases of gauge theories. We will here rule out asymptotic safety for broad classes of supersymmetric theories, including $\mathcal{N} = 1$ cousins of the nonsupersymmetric asymptotically safe theories [1]. Our methods and results do not rely on perturbation theory. In some cases, a hypothetical asymptotically safe UV fixed point would violate unitarity bounds [16], and in other cases it would violate the 4d a-theorem [17–23], which can be explored in the susy context via the connection to 't Hooft anomalies for the superconformal R-symmetry [24–31].

The outline of this paper is as follows. In Section II, we summarize the perturbatively accessible, non-supersymmetric asymptotically safe case [1]. In Section III, we summarize the main susy-based methods that we will use in the following sections. In Section IV, we consider the direct $\mathcal{N} = 1$ cousin of the theory considered in [1], based on $\mathcal{N} = 1$ SQCD above the asymptotic freedom bound, $N_f > 3N_c$. We show that there cannot be a UV-interacting RG fixed point, neither with added gauge singlets Yukawa-coupled to the matter, nor for SQCD without the gauge singlets. In Section V we apply susy-based methods to rule out asymptotically safe UV fixed points for theories with more general gauge group and matter content. We offer our conclusions in Section VI.

II. $\mathcal{N} = 0$ ASYMPTOTIC SAFETY, BRIEF REVIEW OF [1]

Consider a massless theory with $SU(N_c)$ gauge group and N_f, $SU(N_c)$ fundamental, Dirac fermions q_D; we also write them as Weyl fermions q and $\tilde{q}$. The theory has a global $SU(N_f) \times SU(N_f) \times U(1)_B$ symmetry, and we include $N_f \times N_f$ complex scalar fields, which are gauge singlets. The matter content is summarized in Table I. The Lagrangian is

$$L = -\tfrac{1}{4g^2} \text{Tr}\, F^{\mu\nu} F_{\mu\nu} + \text{Tr}\left(\bar{q}_D\, i\slashed{D}\, q_D\right) + \text{Tr}\left(\partial_\mu H^\dagger\, \partial^\mu H\right)$$
$$+ y\, \text{Tr}\left(\tilde{q} H q + \text{h.c.}\right) - h\, \text{Tr}\left(H^\dagger H\right)^2 - v\,(\text{Tr}\, H^\dagger H)^2\,, \tag{2.1}$$

and Tr indicates the appropriate trace over the suppressed color and flavor indices. The classical theory is scale invariant with four marginal couplings: the gauge coupling g, the Yukawa coupling y, the quartic scalar couplings h and the double-trace scalar coupling v. In the quantum theory, it is convenient to introduce

$$\alpha_g = \frac{g^2\, N_c}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_c}{(4\pi)^2}\,, \quad \alpha_h = \frac{h\, N_f}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\, N_f^2}{(4\pi)^2}\,, \tag{2.2}$$

with appropriate powers of N_c and N_f in the normalization to allow for the Veneziano limit of large N_c and N_f, holding fixed

$$x \equiv \frac{N_f}{N_c} \equiv \frac{11}{2} + \epsilon. \tag{2.3}$$

The one-loop beta function is asymptotically free for $\epsilon < 0$, and infrared free for $\epsilon > 0$. In this notation, the usual Banks-Zaks [32] limit is ϵ infinitesimally negative, whereas [1] instead considers ϵ infinitesimally positive.

The relevant beta functions $\beta_i(\alpha_g, \alpha_y, \alpha_h, \alpha_v) \equiv \partial_t \alpha_i$ for each coupling $i = (g, y, h, v)$ of the theory (2.1) have been obtained in [33] in dimensional regularization using [34–37]. The point $\alpha_i = 0$ is IR attractive, since none of the couplings are asymptotically free. As shown in [1], the beta functions vanish at non-zero couplings, compatible with classical and

Fields	$[SU(N_c)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$
A_μ	Adj	1	1	0
q	$\square$	$\bar{\square}$	1	1
$\tilde{q}$	$\bar{\square}$	1	$\square$	-1
H	1	$\square$	$\bar{\square}$	0

Table I: Field content of the $\mathcal{N} = 0$ theory and field. The A_μ are the gauge fields, q and $\tilde{q}$ are Weyl spinors in the $(1/2, 0)$ Lorentz representation, and the H are scalars.

quantum scalar potential stability [5], given by

$$\alpha_g^* = \tfrac{26}{57}\,\epsilon + \tfrac{23(75245-13068\sqrt{23})}{370386}\,\epsilon^2 + \mathcal{O}(\epsilon^3)$$

$$\alpha_y^* = \tfrac{4}{19}\,\epsilon + \left(\tfrac{43549}{20577} - \tfrac{2300\sqrt{23}}{6859}\right)\,\epsilon^2 + \mathcal{O}(\epsilon^3)$$

$$\alpha_h^* = \tfrac{\sqrt{23}-1}{19}\,\epsilon + \mathcal{O}(\epsilon^2)\,,$$

$$\alpha_v^* = \tfrac{1}{19}(-2\sqrt{23} + \sqrt{20+6\sqrt{23}})\,\epsilon + \mathcal{O}(\epsilon^2)\,. \tag{2.4}$$

The phase diagram of the theory was established in [1] at the next-to-leading order accuracy and extended to the next-to-next leading order in [5].

See Fig. 1 of [5] for the RG trajectories of the couplings $(\alpha_g, \alpha_y, \alpha_h, \alpha_v)$; the beta functions are all positive for $\alpha_{i=g,y,h,v} < \alpha_i^*$. The interesting RG trajectory goes from the interacting fixed point at $\alpha_i = \alpha_i^*$ in the UV, and ends at the free theory, $\alpha_i = 0$ in the IR. Along this one-dimensional line of physics, in the 4d α_i space, the Yukawa and scalar quartic couplings are all determined in terms of the running gauge coupling. This dynamical relation among the couplings is dictated by the dimension of the critical surface. In the UV, the gauge coupling approaches the interacting, asymptotically safe, UV fixed point by a power-law in the RG scale

$$\lim_{\mu/\mu_0 \to \infty} \alpha_g(\mu) \to \alpha_g^* + (\alpha_g(\mu_0) - \alpha_g^*)\left(\frac{\mu}{\mu_0}\right)^{-\frac{104}{171}\epsilon^2 + \mathcal{O}(\epsilon^3)}, \tag{2.5}$$

(see [5] for the all-μ running, in terms of the Lambert function $W(\mu)$).

It would interesting to extend the results beyond the perturbative regime via, for example, first principle lattice simulations [38–42]. An alternative limit is QCD for fixed N_c and large N_f where, for the theory without scalars, at leading order $1/N_f$, an UV asymptotically safe fixed point seems also to appear [1, 2]. Supersymmetry provides tools to explore beyond perturbation theory, as we will discuss in the following sections.

III. SUPERSYMMETRIC RG FIXED POINTS AND RG FLOWS

We focus on theories in $d = 4$ spacetime dimensions, with $\mathcal{N} = 1$ supersymmetry. A RG fixed point is an $\mathcal{N} = 1$ superconformal field theory (SCFT), which necessarily has a conserved $U(1)_R$ global symmetry. The $U(1)_R$ current is in the same supermultiplet [43] as the energy-momentum tensor and the supercharge currents; this leads to many useful exact relations. For a unitary theory, the operators form unitary representations of the superconformal group, which implies that operator dimensions have various lower bounds. For example, regardless of supersymmetry, all gauge invariant spin $j = \bar{j} = 0$ operators have the lower bound (generators act with implicit commutators) [16] (see also e.g. [44])

$$D(\mathcal{O}) \geq 1, \qquad D(\mathcal{O}) = 1 \leftrightarrow P_\mu P^\mu(\mathcal{O}) = 0, \tag{3.1}$$

so the bound is saturated if and only if the operator $\mathcal{O}$ is a decoupled, free field. Chiral primary operators have dimension, D, and superconformal $U(1)_R$ charge, R, related by

$$D(\mathcal{O}) = \frac{3}{2}R(\mathcal{O}). \tag{3.2}$$

In particular, using (3.2) for the matter chiral superfields Q_i of a supersymmetric gauge theory relates the matter anomalous dimensions γ_i to their superconformal $U(1)_R$ charge.

$$D(Q_i) \equiv 1 + \frac{1}{2}\gamma_i(g) = \frac{3}{2}R(Q_i) \equiv \frac{3}{2}R_i. \tag{3.3}$$

The conformal anomaly a of the SCFT is exactly given by the superconformal $U(1)_R$ 't Hooft anomalies [24, 25] (we rescale the overall normalization factor for convenience)

$$a(R) = 3\mathrm{Tr}U(1)_R^3 - \mathrm{Tr}U(1)_R. \tag{3.4}$$

For a gauge theory with gauge group, G, and matter fields Q_i, in representations r_i of G, the 't Hooft anomalies evaluate to

$$a(R_i) = 2|G| + \sum_i |r_i|a_1(R_i), \tag{3.5}$$

where $|G| = |r_{\mathrm{Adjoint}}|$ is the number of generators of the gauge group, $|r_i|$ is the dimension of the r_i representation, $R_i \equiv R(Q_i)$ is the $U(1)_R$ charge of Q_i, and we define the function

$$a_1(R) \equiv 3(R-1)^3 - (R-1) . \tag{3.6}$$

Among all possible, conserved $U(1)_R$ symmetries, the superconformal $U(1)_R$ is that which maximizes $a(R)$ [26]. For example, for a chiral superfield X of charge $R(X) = R$ (so $R(\psi_X) = R - 1$), the function is $a(R) = a_1(R)$ in (3.6). The function $a_1(R)$ has a local maximum at the free-field value, $R = \frac{2}{3}$, and a local minimum at $R = \frac{4}{3}$. Indeed $a_1(R) = -a_1(2 - R)$, so $a_1(R = 1) = 0$, fitting with massive operators contributing $a = 0$. Note also that $a_1(R)$ is below the local maximum, $a_1(R) < a_1(R = 2/3)$, for all R in the range $R < 5/3$ (see [48] for a related conjectured phase diagnostic). For unconstrained, i.e. free chiral superfields, we maximize the function (3.6) to get $R_* = 2/3$, the free-field value of the R-charge, corresponding to $D(X) = 1$. With interactions, we maximize $a(R)$ subject to the constraint that the interactions do not violate the R-symmetry. Accidental symmetries affect a-maximization [28](see also e.g. [29]), which if present leads to a larger value of a.

Away from a RG fixed point, the beta functions are proportional to how the couplings break the superconformal $U(1)_R$. The gauge coupling beta function is proportional to the ABJ triangle anomaly of the $U(1)_R$ current with two G gauge fields, i.e. Tr $G^2U(1)_R$:

$$\beta(g) = -\frac{3g^3}{16\pi^2}f(g^2)\mathrm{Tr}\ G^2U(1)_R, \qquad \mathrm{Tr}\ G^2U(1)_R = T(G) + \sum_i T(r_i)(R_i - 1). \tag{3.7}$$

Our normalization for the quadratic Casimir of the adjoint $T(G)$ is $T(SU(N_c)) = N_c$, so the fundamental representation of $SU(N)$ has $T(r_{\mathrm{fund}}) = \frac{1}{2}$. The function $f(g^2) = 1 + \mathcal{O}(g^2)$ is scheme dependent (and presumed positive). Using (3.3) gives the statement of the NSVZ exact beta function (NSVZ also have a favored, specific scheme choice for $f(g^2)$) [45]:

$$\beta(8\pi^2 g^{-2}) = f(g^2)(3T(G) - \sum_i T(r_i)(1 - \gamma_i(g))). \tag{3.8}$$

For superpotential terms W_y, the holomorphic coupling y prefactor has beta function

$$\beta(y) = \frac{3}{2}y(R(W_y) - 2). \tag{3.9}$$

Fields	$[SU(N_c)]$	$SU_L(N_f)$	$SU_R(N_f)$	$U_V(1)$	$U(1)_R$
W_α	Adj	$\mathbf{1}$	$\mathbf{1}$	0	1
Q	$\square$	$\overline{\square}$	$\mathbf{1}$	1	$1 - \frac{N_c}{N_f}$
$\widetilde{Q}$	$\overline{\square}$	$\mathbf{1}$	$\square$	-1	$1 - \frac{N_c}{N_f}$
H	$\mathbf{1}$	$\square$	$\overline{\square}$	0	$2\frac{N_c}{N_f}$

Table II: The $\mathcal{N} = 1$ superfield content, cousins of the theory (I). W_α is the gauge vector superfield, Q and $\widetilde{Q}$ are the matter chiral superfields, and H are gauge singlet chiral superfields.

IV.   SUPERSYMMETRIC QCD WITH $N_f > 3N_c$ IS UNSAFE

As a first class of examples, we consider the $\mathcal{N} = 1$ cousin of the theory (2.1), with superfield content and quantum symmetries summarized in Table II. The superpotential is:

$$W = y\,\mathrm{Tr}\,QH\widetilde{Q} \;, \tag{4.1}$$

with y the Yukawa coupling and Tr contracts the implicit gauge and flavor indices. The quartic in H interactions in (2.1) are incompatible with holomorphy of the superpotential (and the $SU(N_f) \times SU(N_f)$ global symmetry does not allow for a holomorphic variant).

Taking N_c and N_f large, and properly rescaling the couplings as

$$\alpha_g \equiv \frac{g^2 N_c}{(4\pi)^2}\,, \qquad \alpha_y \equiv \frac{y^2 N_c}{(4\pi)^2}\,, \tag{4.2}$$

the two loops beta functions are, dropping terms subleading in $1/N_c$,

$$\beta(\alpha_g) \approx -2\alpha_g^2 \left[3 - \frac{N_f}{N_c} + \left(6 - 4\frac{N_f}{N_c} \right)\alpha_g + 2\frac{N_f^2}{N_c^2}\alpha_y + \mathcal{O}(\alpha^2) \right] \;,$$

$$\beta(\alpha_y) \approx 2\alpha_y \left[\left(2\frac{N_f}{N_c} + 1 \right)\alpha_y - 2\alpha_g + \mathcal{O}(\alpha^2) \right] \;. \tag{4.3}$$

To connect with (3.8) and (3.9) note that they give (using (3.3), and $R(W) = R(H)+2R(Q)$)

$$\beta(\alpha_g) = -2\alpha_g^2 f(\alpha_g) \left(3 - \frac{N_f}{N_c}(1 - \gamma_Q) \right), \qquad \beta(\alpha_y) = \alpha_y(\gamma_H + 2\gamma_Q), \tag{4.4}$$

and to the relevant order (dropping $\mathcal{O}(\alpha_{g,y}^2)$ and $\mathcal{O}(1/N_c)$) we have

$$f(\alpha_g) \approx 1 + 2\alpha_g, \qquad \gamma_Q \approx -2\alpha_g + 2\frac{N_f}{N_c}\alpha_y, \quad \gamma_H \approx 2\alpha_y \tag{4.5}$$

For $N_f > 3N_c$, the theory is not asymptotically free, and $\alpha_g = \alpha_y = 0$ is IR-attractive. We first note that there cannot be a perturbative, interacting, UV-safe fixed point. Define $\epsilon \equiv N_f/N_c - 3$, with N_f and N_c such that $0 < \epsilon \ll 1$. The condition $\beta(\alpha_y) = 0$ then gives $\alpha_y \approx \frac{2}{7}\alpha_g$, and then $\beta(\alpha_g)$, to leading order in small ϵ, is

$$\beta(\alpha_g) \approx 2\alpha_g^2 \left[\epsilon + \frac{6}{7}\alpha_g \right] \;, \tag{4.6}$$

so the relative sign is such that, for positive ϵ, $\beta(\alpha_g) \neq 0$ unless $\alpha_g = 0$.

We now argue, including at the non-perturbative level, that there cannot be a UV-safe interacting SCFT. We assume that the superconformal $U(1)_R$ is not emergent or accidental, in which case it must be the anomaly free R-symmetry that is preserved by the superpotential, i.e. the $U(1)_R$ given in Table II; this ensures that the beta functions (4.4) vanish. The dimension of the operators H are then determined to be

$$D(H) = \frac{3}{2}R(H) = 3\frac{N_c}{N_f} \ . \tag{4.7}$$

These statements are all correct for the *IR* SCFT fixed point when $N_f < 3N_c$. But, for $N_f > 3N_c$, (4.7) would violate the unitarity bound (3.1). This is impossible, since the original theory is unitary. There thus cannot be an interacting UV SCFT for $N_f > 3N_c$.

It follows from Seiberg duality [14] that the $N_f > 3N_c$ theory can actually be UV-completed to an asymptotically *free* dual, rather than an interacting, UV-safe, SCFT.

A potential loophole in the above argument is that apparent unitarity bound violations mean that the corresponding field – in this case H – is instead a free, decoupled field. So we set $\alpha_y = 0$, which is equivalent to considering SQCD without the H singlets. We now argue that SQCD without the H fields also cannot have a UV safe RG fixed point for $N_f > 3N_c$. Consider first the perturbative regime, $\epsilon \equiv N_f/N_c - 3$, with $0 < \epsilon \ll 1$. Taking $\alpha_y = 0$ in (4.3) gives

$$\beta(\alpha_g) \approx 2\alpha_g^2 \left[\epsilon + 6\alpha_g \right] \ ; \tag{4.8}$$

so $\beta(\alpha_g) \neq 0$ for $\epsilon > 0$ and $\alpha_g \neq 0$, i.e. there is no perturbative UV-safe fixed point.

We now rule out asymptotic safety beyond perturbation theory. First note that, for $\alpha_g \neq 0$, having $\beta(\alpha_g) = 0$ requires that the superconformal $U(1)_R$ be that in Table II, so

$$D_{SCFT}(Q) = D_{SCFT}(\widetilde{Q}) \equiv 1 + \frac{1}{2}\gamma_Q = \frac{3}{2}R_{SCFT}(Q) = \frac{3}{2}\left(1 - \frac{N_c}{N_f}\right). \tag{4.9}$$

The gauge invariant chiral operators, the mesons $\mathcal{M} = Q\tilde{Q}$ and baryons $\mathcal{B} = Q^{N_c}$ have

$$D_{SCFT}(\mathcal{M}) = \frac{3}{2}R_{SCFT}(\mathcal{M}) = 3\frac{N_f - N_c}{N_f} \ ,$$

$$D_{SCFT}(\mathcal{B}) = D_{SCFT}(\widetilde{\mathcal{B}}) = \frac{3}{2}R_{SCFT}(\mathcal{B}) = \frac{3}{2}N_c\frac{N_f - N_c}{N_f} \ . \tag{4.10}$$

These expressions are indeed correct for the IR fixed point of SQCD in the conformal window [14] $\frac{3}{2}N_c < N_f < 3N_c$. Extrapolating to the hypothetical, UV fixed point for $N_f > 3N_c$, these expressions would apply, and the operators would satisfy the unitarity bound (3.1).

We therefore rule out a hypothetical, interacting UV fixed point for $N_f > 3N_c$ SQCD on different grounds, by noting that it would violate the 4d a-theorem [17, 22, 23],

$$\Delta a \equiv a_{UV} - a_{IR} > 0. \tag{4.11}$$

If the hypothetical UV-safe fixed point exists, its a would be given by (3.5) with the interacting, anomaly free $U(1)_R$, i.e. $R_{SCFT}(Q) = (N_f - N_c)/N_f$,

$$a_{\text{hypothetical UV}} = a_{\text{SCFT}} = 2(N_c^2 - 1) + 2N_f N_c a_1(R = 1 - \frac{N_c}{N_f}) \ , \tag{4.12}$$

with $a_1(R)$ the function (3.6). There would be a RG flow from the hypothetical UV SCFT to the IR-free theory with $\alpha_g = 0$ and thus $R(Q) = 2/3$:

$$a_{\text{IR}} = a_{\text{free}} = 2(N_c^2 - 1) + 2N_f N_c a_1(R = 2/3) = 2(N_c^2 - 1) + \frac{4}{9} N_f N_c . \qquad (4.13)$$

So this RG flow would violate the a-theorem (4.11):

$$a_{\text{UV-safe}} - a_{\text{IR-free}} = 2N_c N_f \left(a_1(R = 1 - \frac{N_c}{N_f}) - a_1(R = 2/3) \right) < 0, \qquad (4.14)$$

where the inequality is evident from graphing $a_1(R)$ in (3.6) since, for $3N_c < N_f < \infty$, the R-charge $R(Q) = 1 - \frac{N_c}{N_f}$ is in the range $\frac{2}{3} < R(Q) < 1$, and a_1 in that range is below its local maximum $a_1(R = 2/3)$. Given the wrong sign (4.14), we conclude that there cannot be an interacting, UV safe fixed point. Instead, IR-free electric SQCD theory can be UV-completed to an asymptotically free magnetic dual. For SQCD in the conformal window, the identification of the endpoints is opposite from that in (4.14) (free in the UV and interacting in the IR). Both those RG flows of course do satisfy the a-theorem [24, 25].

V.   SUSY THEORIES WITH GENERAL GAUGE GROUP AND MATTER

We first consider susy gauge theories without superpotential terms, $W_{\text{tree}} = 0$. The exact beta function for the gauge coupling is as in (3.7), and the condition $\beta(g) = 0$ is equivalent to the condition that the superconformal $R_{\text{SCFT}}(Q_i) \equiv R_{\text{SCFT},i}$ is anomaly free:

$$\beta(\alpha_g) = 0 \quad \leftrightarrow \quad T(G) + \sum_i T(r_i)(R_i - 1) = 0 \quad \leftrightarrow \quad 3T(G) - \sum_i T(r_i)(1 - \gamma_{Q_i}) = 0. \quad (5.1)$$

$$\text{So not asymptotically free has:} \quad 3T(G) - \sum_i T(r_i) < 0. \qquad (5.2)$$

It is then impossible to satisfy (5.1) perturbatively, since all perturbative γ_{Q_i} are negative.

Generalizing the argument of the previous section, we can moreover rule out UV-safe SCFTs in the range (5.2), without relying on perturbation theory. The RG flow from the hypothetical, asymptotically safe SCFT in the UV, to the free $\alpha_g = 0$ theory in the IR, would violate the a-theorem. Using (3.5), the hypothetical flow has

$$\Delta a = a_{\text{UV-safe}} - a_{\text{IR-free}} = \sum_{\text{all matter } Q_i} |r_i| \left(a_1(R_{\text{SCFT},i}) - a_1(R_i = 2/3) \right). \qquad (5.3)$$

The general expression for $R_{\text{SCFT},i}$ follows from a-maximization [26], i.e. maximizing $a_1(R_{\text{SCFT},i})$ over the R_i, subject to the constraint in (5.1). This gives [27],

$$R_{\text{SCFT},i} = 1 - \frac{1}{3} \left(1 + \frac{\lambda T(r_i)}{|r_i|} \right)^{1/2} , \qquad (5.4)$$

where the Lagrange multiplier is determined via (5.1). In the asymptotically free case, this yields $\lambda = (g_*^2/2\pi^2) + \mathcal{O}(g_*^4)$ (the higher order terms are scheme dependent) [27, 30, 31]. In

the non-asymptotically free case (5.2), on the other hand, the constraint leads to $\lambda < 0$, and all $R_{SCFT,i}$ would be in the range $2/3 < R_i < 1$. It is then clear from the graph of $a_1(R)$ that, since $R_i < 5/3$, every term in (5.3) is negative, $a_1(R_{SCFT,i}) - a_1(R_i = 2/3) < 0$, the flow from the hypothetical, UV-safe fixed point would violate the a-theorem. So susy gauge theories with $W = 0$ cannot be interacting UV-safe, and IR free, without some new element (for example, some accidental symmetry in the interacting UV theory).

We now consider adding superpotential terms. The upshot of a-maximization is that the microscopic fields have superconformal R-charges given by [30]

$$R(Q_i) = 1 - \frac{\epsilon_i}{3}\sqrt{1 - 2\gamma_i^{(1)}(\lambda)}, \tag{5.5}$$

where $\epsilon_i = \pm 1$ (see [30, 49] for curiosities related to ϵ_i sign changes in RG flows) and $\gamma_i^{(1)}$ are linear in the λ and related to the one-loop anomalous dimensions. The values of $R_{SCFT}(Q_i)$ are obtained from (5.5) by solving for the λ_{SCFT} such that all R-charge conservation constraints are satisfied, i.e. all gauge groups G have $\text{Tr}G^2 U(1)_R = 0$ and all W_y have $R(W_y) = 2$, i.e. all beta functions (3.7) and (3.9) vanish. Note that all (5.5) yield $R_{SCFT,i} < 4/3$, and thus all $a_1(R = R_{SCFT,i}) < a_1(R = 2/3)$. The RG flow from a hypothetical UV-safe SCFT, to the IR-free theory, would have Δa given by (5.3), which again would violate the a-theorem because every term in the sum has the wrong sign.

Also, as in the examples (4.1), gauge singlets H coupled to composite operators $\mathcal{O}$, of classical dimension $D_{cl}(\mathcal{O}) = 2$, leads to a unitarity bound problem for hypothetical UV-safe SCFTs. Because the theories are not asymptotically free, the condition (5.1) leads to $R_{SCFT}(\mathcal{O}) > R_{free}(\mathcal{O})$, and then $R(W) = 2$ requires $R(H) < 2/3$; then $D_{UV-safe} = \frac{3}{2}R(H) < 1$, violating the unitarity bound. The H fields must instead remain free; this fits with the fact that there is no interacting, UV-safe SCFT after all.

Note that $\mathcal{N} = 4$ supersymmetric theories, deformed to $\mathcal{N} = 1$ by making the cubic superpotential coupling, y, differ from the gauge coupling, g, have RG flows somewhat similar to, but crucially different from, asymptotic safety. Setting either $y = 0$ or $g = 0$, the other coupling is *marginally* irrelevant. With both $y \neq 0$ and $g \neq 0$, there are RG flows with an IR-attractive line of interacting SCFTs for all $g = y$; the UV limit of the flows for $g \neq y$ has Landau poles. The adjoint chiral superfields $\Phi_{i=1,2,3}$ have $R(\Phi_i) = 2/3$ for all $g = y$ SCFTs (the Φ_i are not free, because they are not gauge invariant). These examples do not contradict our general arguments. Their one-loop beta function in (5.2) are zero, and that there is a line of fixed points, rather than a RG flow, between the interacting ($g = y \neq 0$) and free ($g = y = 0$) theories, all with equal value of a. And there is no UV SCFT, only IR SCFTs. There are many similar $\mathcal{N} = 1$ SCFT examples, along the lines of [47].

Another example of a non-asymptotically free theory is $\mathcal{N} = 1$ gauge theory with the three adjoints of $\mathcal{N} = 4$, plus N_f flavors of matter in the fundamental (as is sometimes considered in the context of AdS/CFT, via added D7 branes). By our general argument, such theories cannot[1] have a UV-safe SCFT without violating the a-theorem.

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[1] This fits with the dual gravity analysis in [50]. KI thanks David Mateos for pointing out this reference.

VI. CONCLUSIONS

We investigated the nonperturbative gauge dynamics of $\mathcal{N} = 1$ super QCD both with, and without, gauge singlet (dual meson) fields H, in the N_f, N_c regime where asymptotic freedom is lost. Unlike the non-supersymmetric case [1], the H fields do not help to achieve asymptotic safety. Instead, the H fields would necessarily violate a unitarity bound in a hypothetical, asymptotically safe SCFT unless their Yukawa coupling is zero and they are decoupled. We showed that the theory without the H fields also cannot have a UV-safe SCFT, because it would violate the a-theorem. We used a-maximization to show that the same issues arise for general gauge groups and matter content.

We arrive at the interesting conclusion that there is a fundamental obstacle to supersymmetric asymptotic safety. Any sensible, UV completion of unsafe IR theories must include many additional degrees of freedom, e.g. those of an unknown, asymptotically free dual.

Note added in revised version: We would like to thank Steven Martin and James Wells for bringing their relevant paper [51] to our attention. Although there is considerable overlap, our paper contains additional methods – for example, using a-maximization, which had not yet been developed at the time [51] was written. Intriguingly, [51] suggested a possible way to construct superconformal UV fixed point theories via superpotential terms. To quote an example from [51], consider $SU(N_c)$ gauge theory with N_f fundamental flavor chiral superfields Q, $\widetilde{Q}$, two adjoints chiral superfields A_1 and A_2, and a superpotential $W = A_1 Q\widetilde{Q} + A_1^3$. The theory is IR free for $N_f > N_c$. A hypothetical UV interacting fixed point would have $R(A_1) = R(Q) = R(\widetilde{Q}) = 2/3$, and $R(A_2) = (N_f + N_c)/3N_c$, which would satisfy $a_{UV} > a_{IR}$ if $R(A_2) > 5/3$, i.e. if $N_f > 4N_c$. We subjected this example to a few additional consistency conditions, including those in this paper and also e.g. verifying that a/c satisfies the inequalities of [52]. We have not yet found any inconsistency to definitively rule out the hypothetical UV-safe SCFT for $N_f > 4N_c$. We note however that replacing the adjoint A_1 with a gauge singlet S_1 would *not* give an interacting SCFT: in that case, since S_1 is a gauge invariant chiral operator, $R(S_1) = 2/3$ would imply $\Delta(S_1) = 1$ and then S_1 would necessarily be a free-field, incompatible with the superpotential coupling having an interacting fixed point of its beta function.

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